MEAN FITNESS AND EVOLUTION

• **Wright’s adaptive topography** (or "landscape")

– Sewall Wright showed that evolution by natural selection of gene frequencies is intimately connected with the population mean fitness.

– The MATH... (Caveat: Derivation assumes fitnesses w_{AA}, w_{Aa}, w_{aa} are <u>constant.)</u>

• Begin with the basic formula for mean fitness for a single locus with two alleles:

$$\overline{w} = p^2 w_{AA} + 2pqw_{Aa} + q^2 w_{aa} \tag{1}$$

• Take derivative of Equation (1) with respect to p (note: $dq/dp = -1$):

$$\frac{d\overline{w}}{dp} = 2\left(pw_{AA} + qw_{Aa} - pw_{Aa} - qw_{aa}\right) = 2\left[\left(pw_{AA} + qw_{Aa}\right) - \left(pw_{Aa} + qw_{aa}\right)\right] \ . \tag{2}$$

• We already know the rate of change in an allele's frequency under selection is

$$\Delta p = pq\frac{\overline{w}_A - \overline{w}_a}{\overline{w}} = \frac{pq}{\overline{w}}\left(\overline{w}_A - \overline{w}_a\right) = \frac{pq}{\overline{w}}\left[\left(pw_{AA} + qw_{Aa}\right) - \left(pw_{Aa} + qw_{aa}\right)\right] \tag{3}$$

• Notice that the expression in brackets on the right hand side of Equation (3) is ½ of the right-hand side of (2). Therefore

$$\Delta p = \frac{pq}{\overline{w}}\left(\frac{1}{2}\frac{d\overline{w}}{dp}\right) = \frac{pq}{2}\left(\frac{1}{\overline{w}}\frac{d\overline{w}}{dp}\right) = \frac{pq}{2}\left(\frac{d\ln\overline{w}}{dp}\right) \ . \tag{4}$$

– Main Result: $\Delta p = \frac{pq}{2}\left(\frac{d\ln\overline{w}}{dp}\right)$

– <u>Conclude</u>: Rate of evolution by natural selection is determined by
1) extent of genetic variation
2) the effect that a change in allele freq. will have on the population's mean fitness, $\overline{w}$

– The “**Adaptive topography**”

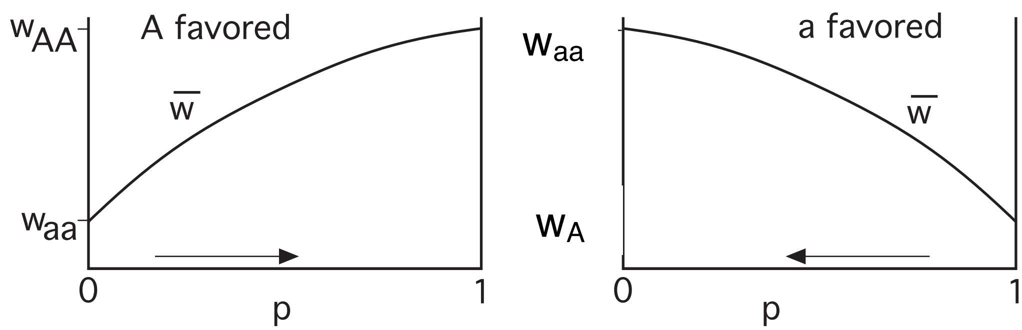
• Metaphor (often used improperly) for how populations evolve due to selection;

• Math shows that allele frequencies always change such that $\overline{w}$ increases.

• Consequently, selection “moves” the population uphill on a graph of $\overline{w}$ vs. p:

– <u>No over- or underdominance</u>

- Mean fitness is at a maximum at $p = 0$ or $p = 1$, depending on which allele is favored:

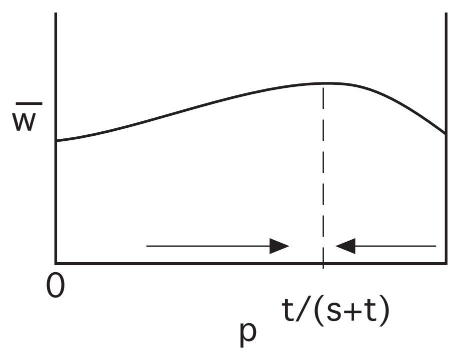


– <u>Overdominance</u>

$$\begin{matrix} w_{AA} & w_{Aa} & w_{aa} \\ 1-s: & 1 & :1-t \end{matrix}$$

- Mean fitness has a peak at
$$\hat{p} = \frac{t}{s+t}$$

- At equilibrium, selection tries to increase $\overline{w}$, but segregation opposes this.



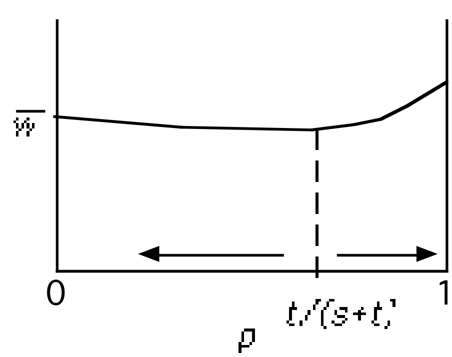
  – With random mating:
  $$\hat{\overline{w}} = 1 - \hat{p}^2 s - \hat{q}^2 t$$
  – IF population were 100% heterozygous, then $\overline{w} = 1$
  – presence of *AA* homozygotes decrease mean fitness by $\hat{p}^2 s \approx 0.11$
  – While if $t = 1$ (i.e., *aa* is lethal), then *aa* decreases fitness by $\hat{q}^2 t \approx 0.01$
  - Combined effects of *AA*, *aa* called "<u>segregation</u> <u>load</u>" = "decrease in maximal mean fitness due to segregation"

– <u>Underdominance</u>

- $\overline{w}$ has two maxima, at $p = 0$ and $p = 1$.

- Selection is "blind" to which equilibrium it moves toward; It only moves uphill.

- Consequence: Depending on history, a population may never reach a state of globally maximal (optimal) fitness!!



– Digression: the Adaptive Trampoline?

• **Fisher's "Fundamental" Theorem of Natural Selection**

– **Haploid** (**asexual**) version:

• $\overline{w} = pw_A + qw_a$

• $\overline{w}' = p'w_A + q'w_a = \left(p\frac{w_A}{\overline{w}}\right)w_A + \left(q\frac{w_a}{\overline{w}}\right)w_a = \frac{pw_A^2 + qw_a^2}{\overline{w}} = \frac{\overline{w^2}}{\overline{w}}$

• Thus, $\Delta\overline{w} = \overline{w}' - \overline{w} = \frac{\overline{w^2}}{\overline{w}} - \overline{w} = \frac{\overline{w^2} - \overline{w}^2}{\overline{w}} = \frac{\mathrm{Var}(w)}{\overline{w}}$

• Suppose we scaled w_A, w_a so that $\overline{w} = 1$. Then $\Delta\overline{w} = \mathrm{Var}(w)$.
– Variances are never negative, so $\overline{w}$ never decreases.

– **Diploid** derivation & version:

• Another way to write the equation for Δp is

$$\Delta p = p' - p = p\frac{\overline{w}_A}{\overline{w}} - p = p\frac{\overline{w}_A - \overline{w}}{\overline{w}} = p\frac{\alpha_A}{\overline{w}} \tag{5a}$$

• Fisher referred to the quantity $\alpha_A = \overline{w}_A - \overline{w}$ as the average excess of allele A.

• We can likewise write the equation for Δq as

$$\Delta q = q\frac{\overline{w}_a - \overline{w}}{\overline{w}} = q\frac{\alpha_a}{\overline{w}} \tag{5b}$$

where $\alpha_a = \overline{w}_a - \overline{w}$ is the average excess of allele a.

• Equations (5a) and (5b) show us instantly whether p or q is increasing or decreasing: it depends simply on whether the average excess is positive or negative.

• What is the mean relative fitness in the next generation? Using the fact $\Delta q = -\Delta p$ (*Suggested exercise*: show this), we find

$$
\begin{aligned}
\overline{w}' &= p'^2 w_{AA} + 2p'q'w_{Aa} + q'^2 w_{aa} \\
&= (p+\Delta p)^2 w_{AA} + 2(p+\Delta p)(q-\Delta p)w_{Aa} + (q-\Delta p)^2 w_{aa} \\
&= \overline{w} + 2\Delta p\left[pw_{AA} + qw_{Aa} - (pw_{Aa} + qw_{aa})\right] + (\Delta p)^2 (w_{AA} - 2w_{Aa} + w_{aa})
\end{aligned}
\tag{6}
$$

- If differences in fitness are small, allele frequencies will be changing slowly and $(\Delta p)^2$ will be very small compared to Δp. We can therefore neglect terms involving $(\Delta p)^2$ in Equation (6) and approximate the change in $\overline{w}$ as:

$$
\begin{aligned}
\Delta\overline{w} &= \overline{w}' - \overline{w} \\
&\approx 2\Delta p\left[pw_{AA} + qw_{Aa} - (pw_{Aa} + qw_{aa})\right] \\
&\approx 2\Delta p(\overline{w}_A - \overline{w}_a) \\
&\approx 2\Delta p\left[(\overline{w}_A - \overline{w}) - (\overline{w}_a - \overline{w})\right] \\
&\approx 2\Delta p(\overline{w}_A - \overline{w}) + 2\Delta q(\overline{w}_a - \overline{w}) = 2\Delta p\alpha_A + 2\Delta q\alpha_a
\end{aligned}
$$

- Now use Equations (5a) and (5b) by inserting those definitions for Δp and Δq into the above to find:

$$
\Delta\overline{w} \approx 2\left(p\frac{\alpha_A}{\overline{w}}\right)\alpha_A + 2\left(q\frac{\alpha_a}{\overline{w}}\right)\alpha_a = \frac{2\left(p\alpha_A^2 + q\alpha_a^2\right)}{\overline{w}} \quad . \tag{7}
$$

  – The expression in the numerator of (7) is simply the variance in α, the relative excess of fitness, for the two alleles (the factor 2 appears because we are dealing with diploids). This quantity is known as the <u>additive genetic variance</u> of fitness, sometimes denoted V_A.

- Equation (7) can thus be written as $\Delta\overline{w} \approx \frac{\mathrm{Var}(\alpha)}{\overline{w}} = \frac{V_A}{\overline{w}}$.

- We can rescale to appropriate units by dividing through by $\overline{w}$. This expresses $\Delta\overline{w}$ as a proportional change, and standardizes the variance to squared units of $\overline{w}$. The big conclusion is that $\frac{\Delta\overline{w}}{\overline{w}} \approx \frac{V_A}{\overline{w}^2}$.

- This shows Fisher's Fundamental Theorem. (Approximately, that is, which is the best that can be done; it's not exactly true even for a single diploid locus!) As Fisher states it (1958, p. 37):

  "*The rate of increase in fitness of any organism at any time is equal to its genetic variance in fitness at that time*."

- J.F. Crow paraphrases Fisher's words in the more accurate statement:

"*The relative (geometric) rate of increase in mean fitness in any generation is approximately equal to the standardized additive genetic variance of fitness at that time*."

- **Comments on Fitness Maximization**
  - – Selection works to increase $\overline{w}$ under <u>some</u> circumstances
  - – Other evolutionary forces can cause $\overline{w}$ to decrease, even when selection favors increasing $\overline{w}$.