## Selection and Mutation

- Consider cases with no "back mutation" (i.e., no mutation to advantageous types)
- This is often a good approximation to the real world since:
(1) backward mutation is usually 2 or more orders of magnitude smaller than forward mutation.
(2) many more deleterious alleles will be eliminated by selection than are created by mutation so deleterious alleles are rare anyway.
- Note: our focus is on deleterious mutations here since in these cases, selection opposes mutation as an evolutionary force.
(1) Haploid mutation-selection balance:
$\bullet$ Life Cycle: $\underset{p}{\text { zygotes }} \xrightarrow[p^{*}]{\text { selection }} \underset{p^{* *}}{\text { adults }} \xrightarrow{\text { mutation }} \underset{p^{\prime}}{\text { gametes }} \xrightarrow{\text { random union }} \underset{ }{\text { zygotes }}$
- Fitnesses: $w_{A}: w_{a}=1: 1-s$
- $p=$ freq. $A, q=$ freq. $a$
- After selection: $p^{*}=p \frac{w_{A}}{\bar{w}}=p\left(\frac{1}{1-s q}\right)$
- Let $u=A \rightarrow a$ mutation rate (ignore back mutation $a \rightarrow A$ : i.e., $v=0$ ), then

$$
p^{\prime}=p^{* *}=p^{*}(1-u)=\frac{(1-u) p}{1-s q} .
$$

- At equilibrium, $p^{\prime}=p$ and $\hat{q}_{\text {hap }}=1-\hat{p}_{\text {hap }}=u / s$.
- I.e., $\hat{q}_{\text {hap }}=$ ratio of mutation rate to rate of selective elimination.
- Note:
$-u=10^{-6}, s=0.01$ implies $\hat{q}=10^{-4}$
-Recall: not all mutations are deleterious.
- polymorphisms of selectively neutral alleles are maintained by a balance between mutation and random genetic drift.
(2) Diploid mutation-selection balance:

| Genotype | $A A$ | $A a$ | $a a$ |
| ---: | :---: | :---: | :---: |
| Fitness | 1 | $1-h s$ | $1-s$ |

- Using approach analogous to that for haploids (substitute $\bar{w}_{A}$ instead of $w_{A}$, etc.):

$$
p^{*}=p \frac{\bar{w}_{A}}{\bar{w}} \text { and } p^{\prime}=(1-u) p^{*}=(1-u) p \frac{\bar{w}_{A}}{\bar{w}}
$$

- Setting $p^{\prime}=p$, can solve for equilibrium frequency of deleterious $a$ allele $(\hat{q}=1-\hat{p})$ :
- Two cases of interest:
(a) Recessive Mutant: $h=0$
- Solving for equilibrium shows $\hat{q}_{\mathrm{rec}}=\sqrt{u / s}$.
- Note: $u<s$ so $\hat{q}_{\text {rec }}>\hat{q}_{\text {hap }}$ for the same $u, s$.
(b) Partial Dominance: $h>0$
- By ignoring $\hat{q}^{2}$ in the equilibrium equations, find that $\hat{q}_{h} \approx u /(h s)$.
- Approximation fails as $h \rightarrow 0$ (recessive case) .
- Since most affected individuals are heterozygous, $\hat{q}_{h}$ is approximately the ratio of the mutation rate to average selective disadvantage.


## - General Comments

- Fraction of affected individuals is the same in haploids as in diploids
- e.g., haploid affecteds: $\hat{q}_{\text {hap }}=u / s$;
recessive affecteds: $\quad \hat{P}_{a a}=\hat{q}_{\text {rec }}^{2}=(\sqrt{u / s})^{2}=u / s$.
- Can often use observed frequencies and known fitnesses of affected individuals to estimate the mutation rate.


## Mutation Load

- How does mutation impair average population fitness?
- J.B.S. Haldane asked (\& answered) this question in 1937.
- Interest was rekindled (by H. J. Muller) after Hiroshima and Nagasaki bombed using atomic weapons.
- For a single recessive locus, can define this effect as follows:
- Frequency of the affected individuals is $\hat{P}_{a a}=\hat{q}_{\mathrm{rec}}^{2}=(\sqrt{u / s})^{2}=u / s$.
- Fitness lowered by a relative amount $s$ per affected individual
- Total reduction in fitness: $(u / s) \times s=u$
- This is the "mutation load" for a recessive deleterious.
- notice that the mutation load is independent of $s$.
- Similarly, for partially dominant mutations
- Assume $\hat{q}_{h} \ll 1$
- Frequency of affecteds $\approx$ frequency of heterozygotes $=2 \hat{q}_{h}\left(1-\hat{q}_{h}\right) \approx 2 \hat{q}_{h} \approx(2 u) /(h s)$.
- Mutation load (total reduction in fitness): $(2 u / h s) \times h s=2 u$.
- Again, mutation load is independent of $s$.
- Punch Line: Mutation load depends only on mutation rate and not on a mutant's fitness effects (i.e., $s$ ).
- Why is this?
- Highly deleterious mutations equilibrate at low frequencies;
- Mildly deleterious mutations equilibrate at high frequencies;
- Net effect in either case is the same.
- Muller: "One mutation equals one death"
- At equilibrium, each new mutation in a population is offset by the loss of another one due to selection.
- Small selection coefficient means only that the risk of death for an affected individual is smaller, not that the total number of deaths is smaller.
- Either
- many individuals have smaller probabilities of death,
- or few individuals have a high probability of death.
- Ethical dilemma: do medical advances relieve suffering?
- Individual suffering is generally reduced.
- Result is that more individuals suffer mild effects.

