Homework #3 Answer Key

1. (a) $\Delta p = pq\dfrac{s}{1-qs}$ = .0052, .0091

   (b) $\Delta p = pq\dfrac{qs}{1-q^2s}$ = .0050, .0009

   (c) $\Delta p = pq\dfrac{ps}{1-\left(1-p^2\right)s}$ = .0002, .0083

   (d) $\Delta p = pq\dfrac{\left[h(p-q)+q\right]s}{1-q(2ph+q)s}$ = .0031, .0038

   (e) Population genetic architecture (allele frequency): has greatest impact on evolutionary rates in recessive and dominance cases. Rates are high (low) for a rare (common) dominant favorable allele. The opposite is true for a recessive favorable allele. Evolutionary rates are comparatively similar at high and low frequencies for haploids and partial dominance.

   Individual genetic architecture (genetic basis): Haploids had the overall highest rates of evolution for the same strength of selection. In diploids, the rate of evolution at a particular frequency was strongly dependent on whether the favored allele was dominant, recessive, or partially dominant in its effects on individual fitness.

2. (a) $P^*_{AA} = p^2\dfrac{w_{AA}}{\overline{w}} = p^2\dfrac{1}{1-qs} = .5213$; $P^*_{aa} = q^2\dfrac{w_{aa}}{\overline{w}} = q^2\dfrac{1-s}{1-qs} = .0766$,

   $P^*_{Aa} = 1 - P^*_{AA} - P^*_{aa} = .4021$.

   (b) $p^* = P^*_{AA} + P^*_{Aa}/2 = .7224$ but, for example, $P^*_{AA} = .5213 \neq \left(p^*\right)^2 = .5219$. So the post-selection genotype frequencies are *not* in H-W proportions.

   (c) $P^*_{AA} = p^2\dfrac{1}{(1-qs)^2} = .5545$; $P^*_{aa} = q^2\dfrac{(1-s)^2}{(1-qs)^2} = .0652$, $P^*_{Aa} = 2pq\dfrac{(1-s)}{(1-qs)^2} = .3803$.

   (d)

$$p^* = P^*_{AA} + P^*_{Aa}/2 = \left(\frac{p}{1-qs}\right)^2 + \left(\frac{p}{1-qs}\right)\left[\frac{q(1-s)}{1-qs}\right]$$

$$= \left(\frac{p}{1-qs}\right)\left[\frac{p+q(1-s)}{1-qs}\right] = \left(\frac{p}{1-qs}\right)\left(\frac{1-qs}{1-qs}\right) = \left(\frac{p}{1-qs}\right)$$

. Note that

$q^* = 1 - \dfrac{p}{1-qs} = \dfrac{1-qs-p}{1-qs} = \dfrac{q(1-s)}{1-qs}$. So clearly, $P^*_{AA} = \dfrac{p^2}{(1-qs)^2} = \left(p^*\right)^2$,

$P^*_{aa} = \dfrac{q^2(1-s)^2}{(1-qs)^2} = \left(q^*\right)^2$, and $P^*_{Aa} = 1 - P^*_{AA} - P^*_{aa} = 1 - \left(p^*\right)^2 - \left(q^*\right)^2 = 2p^*q^*$.

(e) Because H-W proportions can occur at all life stages with consistent (multiplicative) selective differences among genotypes—a clear violation of the Hardy-Weinberg assumptions.

(f) $\Delta p = pq\frac{\overline{w}_A - \overline{w}_a}{\overline{w}} = pq\frac{(1-qs)-\left[(1-s)(1-qs)\right]}{(1-qs)^2} = pq\frac{1-(1-s)}{1-qs} = pq\frac{s}{1-qs}$.

3. (a) $\Delta p = .51 - .5 = .01 = pq\frac{qs}{1-q^2 s} = (.5)(.5)\frac{(.5)s}{1-(.5)^2 s}$. Solving for s gives $s = .078$.

(b) $\Delta p = .01 = pq\frac{ps}{1-(1-p^2)s} = (.5)(.5)\frac{(.5)s}{1-(.75)s}$. Solving for $s$gives $s = .075$

(c) $\Delta p = .01 = pq\frac{s}{1-qs} = (.5)(.5)\frac{s}{1-(.5)s} \Rightarrow s = .0392$.

(d) Selection is about twice as efficient in haploids compared to diploids.

4. $\overline{w} = \left(1 - \frac{s}{2}\right)^m = .0795$

5. $K_{aa} = -\ln(1-d_{aa}) = -\ln\left(1 - \frac{30}{100}\right) = .357$

standard error of $K_{aa} = \sqrt{\text{var}\left(\frac{d_{aa}}{(1-d_{aa})N}\right)} = .065$

$k_{aa} = \frac{K_{aa}}{2T} = 8.92 \times 10^{-10}$