

MIGRATION

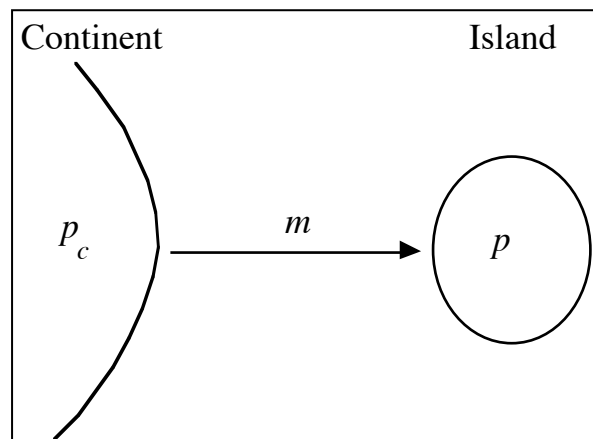
ROLES OF MIGRATION IN EVOLUTION

READING: Hedrick pp. 403-436.

- Introduces novel genetic variation into populations.
- Tends to homogenize gene frequencies in different populations.
- Sets the spatial scale for evolution.
- Opposes local adaptation.
- Migration with an evolutionary impact: Gene Flow
 - Migration introduces *individuals* and *genotypes* (“dispersal”).
 - Migrants have no effect on evolution unless their genes are incorporated into a population.

• A One-Island Model

- The simplest model of migration.
- Two alleles A and a . Let p = frequency of A on island.
- A fraction m of the island gene pool emigrates from the continent where the frequency of A is p_c .
 ⇒ A fraction $(1 - m)$ of alleles on the island originated on the island.
- The continent is too vast to be influenced by migration from the island ⇒ p_c is constant.
- Then the frequency of A on the island changes according to $p' = (1 - m)p + mp_c$.



– At equilibrium, set $p' = p$.

- Solving for p gives $\hat{p} = p_c$

– Rate of approach to equilibrium:

- Rewrite evolutionary equation as

$$p' - \hat{p} = p' - p_c = (1 - m)p + mp_c - p_c = (1 - m)(p - p_c) \\ = (1 - m)(p - \hat{p})$$

– Conclusions

(1) At equilibrium, both populations have the same allele frequencies.

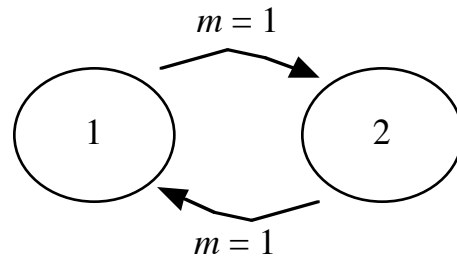
(2) Rate of approach to equilibrium ($\hat{p} = p_c$) is determined by the migration rate m .

• **General Models of Migration**

– Same conclusions as one-island model hold.

– Exceptions, however, do exist

- For example, consider two populations with different allele frequencies that switch locations each generation.
- The populations will obviously never homogenize (because there's no real exchange of genes).



– Remark: Have implicitly assumed gene frequencies differ in different locations.

– How could this be?

- “History.”
- Genetic drift.
- Selection favors different alleles in different locations.

MIGRATION AND DRIFT

- Migration introduces novel genetic variation into local populations.
- Drift removes local genetic variation.

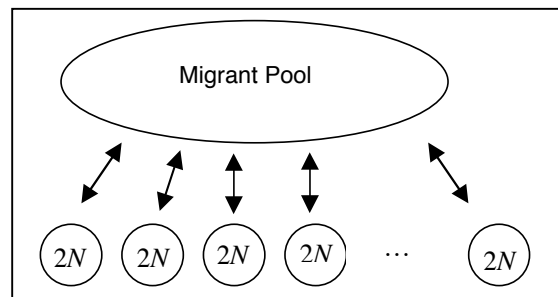
Which for dominates?

One answer...

- **Wright’s “Island Model”**

- Consider a large number of “islands” each with a population of size N ($2N$ alleles per locus)

- Each generation, every island exchanges a fraction m of its gametes with a ∞ -sized “migrant pool” to which all islands contribute gametes.



- Assume infinite-isoalleles model.
- Let $f_t = \text{Pr}(\text{pair or randomly drawn gametes on a typical island are IBD in generation } t) = \text{average within-island homozygosity}$
- By the same logic used when studying mutation-drift balance:

$$f_{t+1} = (1 - m)^2 \left[\frac{1}{2N} + \left(1 - \frac{1}{2N} \right) f_t \right]$$

- At equilibrium, $f_{t+1} = f_t = \hat{f} \approx \frac{1}{1 + 4Nm}$
 - expression resembles that describing diversity maintained by mutation & drift, with $\theta = 4Nu$ replaced by $4Nm$.
- If $4Nm < 1$: Local homozygosity is substantial
 - drift dominates migration
- If $4Nm > 1$: Local diversity (heterozygosity) is substantial
 - migration dominates drift

Note 1

– $4Nm > 1$ same as $2Nm > 1/2$

⇒ Migration dominates drift if at least *one migrant gamete is exchanged every other generation!*

– Conclusion is independent of m , the rate of gene flow. (Why?)

Note 2

– Recall from discussion of F statistics: $\bar{H}_s = \text{Avg}_i(H_{s,i}) \approx 1 - \hat{f}$, since \hat{f} is the average local homozygosity and there is no additional inbreeding

– Also, $H_T = 1 - \text{Pr}(\text{pair of randomly chosen gametes from entire population are IBD}) = 1 - 0 = 1$

$$\Rightarrow F_{ST} = \frac{H_T - \bar{H}_s}{H_T} = \frac{1 - (1 - \hat{f})}{1} = \hat{f} = \frac{1}{1 + M}, \text{ where } M = 4Nm.$$

– Suggests way to estimate rate of migration from F_{ST} :

$$\hat{M} = \frac{1 - F_{ST}}{F_{ST}}.$$

– Careful: estimate requires lots of assumptions (island model, equilibrium, etc.) to be valid.

MIGRATION AND SELECTION

• **One-island model with selection**

- A favored on island.
- a fixed on continent: $p_c = 0$.
- A is dominant.

• Fitnesses on island:

Genotype	AA	Aa	aa
Fitness	1	1	$1 - s$

• Life Cycle: zygotes $\xrightarrow{\text{selection}}$ adults $\xrightarrow{\text{migration}}$ gametes $\xrightarrow{\text{random union}}$ zygotes
 p p^* p^{**} p'

• After selection (before migration): $p^* = p \frac{1}{1 - q^2 s}$

- After migration & reproduction:
$$p' = (1 - m)p^* + m(0) = \frac{p(1 - m)}{1 - q^2s}$$

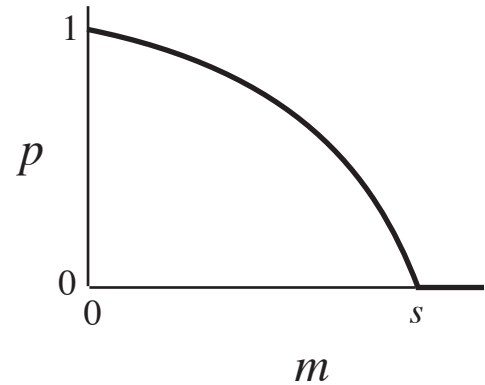
- To find any equilibria, set $p' = p$.

- Solving for p gives $\hat{p} = 1 - \sqrt{m/s}$.

- Require $0 \leq \hat{p} \leq 1$.

- This occurs only when $m < s$.

- Otherwise $\hat{p} = 0$.



- Now assume A is recessive.

- Fitnesses on island:

Genotype	AA	Aa	aa
Fitness	1	1 - s	1 - s

- After selection (before migration):
$$p^* = \frac{p(1 - qs)}{1 - sq(1 + p)}$$

- After migration & reproduction:
$$p' = (1 - m)p^* + m(0) = \frac{p(1 - m)(1 - qs)}{1 - sq(1 + p)}$$

- To find equilibria, set $p' = p$ and solve for p .

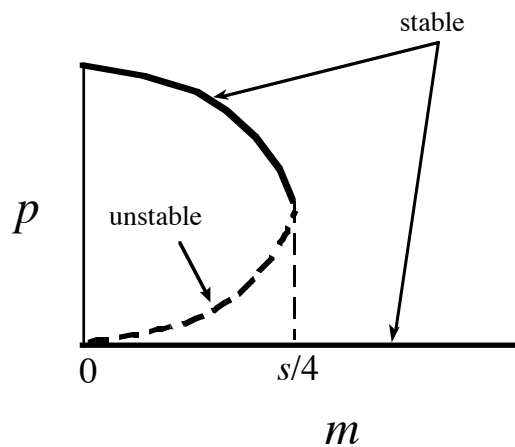
- Get cubic equation for \hat{p} 's (up to 3 possible solutions).

- $\hat{p} = 0$ is always an equilibrium (since $p_c = 0$).

- There are two *polymorphic* equilibria when $s > 4m$ (assuming m is small).

- one equilibrium is stable, the other is unstable.

- Graphically \longrightarrow



- Implications

- If recessive selection is strong enough to maintain A in the face of migration, A will spread only if it's initially sufficiently frequent enough. Otherwise, it will be lost.
- In general, unless locally advantageous allele is completely dominant, it must reach a threshold frequency to persist.
- If an allele persists, it won't be found at a low frequency.
- Historical "accidents" play a role.
 - Identical patches will evolve differently if they differ in initial allele frequency.

• **The Levene Model**

Q: What happens when a population is made up of a group of distinct subpopulation patches, with different selection pressures occurring in each and migration between locations?

A: Depends on geography (population structure).

- Natural populations fall somewhere between the following two extremes:
 - Unrestricted migration.
 - Restricted migration.
- A simple model of unrestricted migration was presented in 1953 by H. Levene.
- Assumptions of Levene's 1953 model:
 - n patches in which different patterns of selection occur.
 - Frequency of A among gametes is p .
 - After fertilization, (diploid) zygotes colonize the different patches (at random).
 - Important: this implies that the zygotes *within* patches are in H-W proportions.
 - i th patch makes up a fraction c_i of the environment.
 - Fitnesses in the i th patch:

Genotype	AA	Aa	aa
Fitness	$w_{AA}(i)$	$w_{Aa}(i)$	$w_{aa}(i)$

- Random mating between patches.
 - Individuals from different localities form a single mating (gamete) pool.
- Why study the Levene model?
 - Captures essential features of spatially subdivided population and is mathematically tractable.
 - Is a reasonable representation of certain natural systems as well.
- Back to model...How many gametes does each patch contribute to the gamete pool?
 - Two extremes:

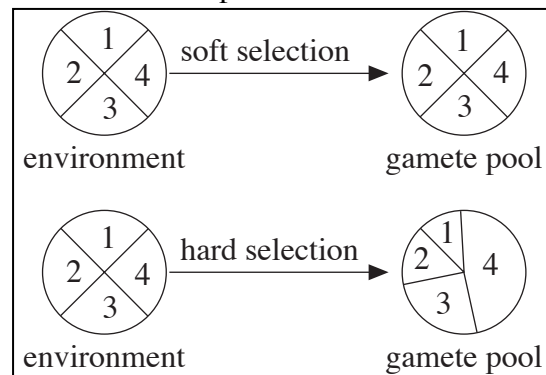
(1) Hard selection (due to Dempster, 1955)

- Patch contributes gametes in proportion to the fraction of survivors.
 - i.e., patches with higher fitness contribute disproportionately more.
- Implies population size is not regulated within patches.

(2) Soft selection

- Each patch contributes fixed number of gametes to the mating pool *regardless* of local fitnesses.
- Number of reproducing adults from each patch is the same from one generation to the next.
- Implies population size is regulated within each patch.

- A schematic comparison between soft and hard selection, assuming $c_1 = c_2 = c_3 = c_4$ and $\bar{w}_4 > \bar{w}_3 > \bar{w}_2 > \bar{w}_1$



– Levene model with hard selection (“constant number of zygotes”):

- Assumes contribution of genotype from patch i to the gamete pool is proportional to its fitness in that patch $[w_{\text{genotype}}(i)] \times$ frequency of i patch in environment (c_i):
 - i.e., total number of survivors of that genotype in patch $i \propto c_i w_{\text{genotype}}(i)$

- Overall fitness of genotype in population is its average fitness over patches:
 - For example, mean fitness of AA: $\bar{w}_{AA} = \sum_{i=1}^n c_i w_{AA}(i)$
 - Likewise for Aa and aa.
- Consider changes in the frequency p of A in the gamete pool.
- $p' = p \frac{p\bar{w}_{AA} + q\bar{w}_{Aa}}{\bar{w}} = p \frac{\bar{w}_A}{\bar{w}}$ where $\bar{w}_A = p\bar{w}_{AA} + q\bar{w}_{Aa}$ and $\bar{w} = p^2\bar{w}_{AA} + 2pq\bar{w}_{Aa} + q^2\bar{w}_{aa}$.
- Looks just like selection with constant fitnesses: $\bar{w}_{AA}, \bar{w}_{Aa}, \bar{w}_{aa}$
- Consequences
 - An allele will spread if it has the highest arithmetic mean fitness across patches.
 - Selection will maintain a stable polymorphism if heterozygotes have the greatest arithmetic mean fitness across patches.

- For example, consider two equally sized patches, $c_1 = c_2 = 0.5$.

Fitness in patch:	AA	Aa	aa
# 1	0	0.75	1
# 2	1	0.75	0
Average:	0.5	0.75	0.5

- Selection maximizes arithmetic mean fitness across environments
- Levene model with soft selection (“constant number of adults”):

– Within each patch, selection operates as usual.

– Fitness in patch i :

	AA	Aa	aa
	w_i	1	v_i

– After selection, frequency of A in patch i is

$$p^*(i) = p \frac{pw_i + q(1)}{p^2w_i + 2pq(1) + q^2v_i} = p \frac{\bar{w}_A(i)}{\bar{w}(i)}$$

– Density regulation occurs independently in each patch.

– Survivors contribute to gamete pool in proportion to the size (= relative proportion of adults) of the patch, c_i :

$$p' = \sum_{i=1}^n c_i p^*(i) = \sum_{i=1}^n c_i p \frac{pw_i + q}{p^2 w_i + 2pq + q^2 v_i}$$

– Equilibrium: set $p' = p$ and solve for p .

- Results in polynomial of degree $2n + 1$ in p .
 \Rightarrow as many as $2n + 1$ equilibria, \hat{p} , are possible!

- Mathematically too difficult to find all these.

– Alternative: protected polymorphism analysis:

- Near $p = 0$, $p' \approx \sum_{i=1}^n \left(\frac{c_i}{v_i} \right) p = p \frac{1}{\tilde{v}}$

where $\tilde{v} = 1 / \left[\sum_{i=1}^n c_i \left(\frac{1}{v_i} \right) \right]$ is the harmonic mean fitness of *aa* homozygotes.

- Note that $p' > p$ (i.e., $\Delta p > 0$) whenever $1/\tilde{v} > 1 \Leftrightarrow \tilde{v} < 1$
 – i.e., whenever the “harmonic mean fitness of *aa* homozygotes” < “mean fitness of heterozygotes”

- Likewise, near $p = 1$, ($q = 0$), $q' > q$ whenever $\tilde{w} < 1$.

- Conclude: protected polymorphism occurs with soft selection whenever there is harmonic mean overdominance in fitness across patches: $\tilde{w} < 1 > \tilde{v}$.

– Bottom line(s) for soft selection

- Harmonic mean fitness across patches is the relevant fitness measure if $p \approx 0$ or 1.
- Turns out, however, that selection maximizes *geometric* mean fitness.

– Hard versus Soft Selection

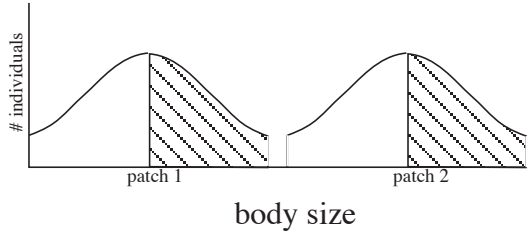
- Conditions exist in which an allele will increase under soft selection but not hard selection.
 - I.e., polymorphisms can be maintained under a broader range of conditions with soft selection versus hard selection.
- Intuitively follows because under soft selection, individuals compete selectively only against “patch-mates”.
 - With hard selection, all compete.

– Mathematically follows because harmonic mean is never larger than the

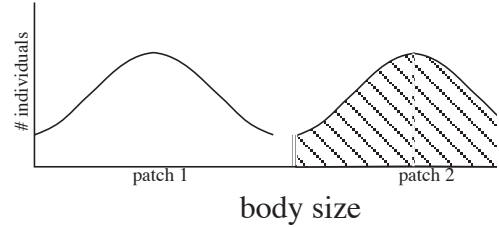
arithmetic mean: $\tilde{v} \leq \bar{v} = \sum_{i=1}^n c_i v_i$.

Q: Why does soft selection seem “hard” (density regulation; intense local competition) while hard selection seems “soft” (little competition; no density regulation)?

A: It all depends on your viewpoint (genetic vs. demographic).



– Soft selection: top 50% in each patch selected.



– Hard selection: top 50% selected (regardless of patch).