

Simulating Cultural Eutrophication without Spurious Oscillations

Note for the “Advanced Exercises”
on the Collection of Eutrophication Exercises
on the Website [for Modeling the Environment](#)

Andrew Ford
May 2002

Introduction

Page 133 of [Toward Global Equilibrium](#) shows a simulation of Jay Anderson’s eutrophication model to depict “cultural eutrophication” over a 200-year time span. The nutrient flowing into the lake is 0.01 mg/liter at the start of the simulation, and the inflow grows exponentially at the rate of 2%/year. The first 150 years of the simulation show “cultural” or “accelerated” eutrophication. Specifically, the biomass concentration increases by around six fold; the detritus concentration increases by around ten fold; and the average oxygen concentration in the lake declines by one third.

After the 150th year, however, the model shows highly unstable oscillations. Biomass, for example, increases much more rapidly than suggested by the previous trend line. Then we see a precipitous drop in the biomass concentration to below the trend line, followed by an even more abrupt increase, followed by another decline. Similar oscillations appear in the detritus concentration and in the average oxygen concentration.

These oscillations are NOT explained by Jay Anderson, so I suspect they are spurious. I have checked whether the oscillations are due to numerical inaccuracies by cutting DT in half and repeating the simulation. The oscillations reappear in the same form, so we know they are not due to numerical instabilities.

I believe that the oscillations arise from the particular combination of stocks and flows that Anderson used. I believe his model is prone to spurious instability, and this note explains how his general assumptions could be represented with a different combination of stocks, flows and converters. The goal is to reproduce the general pattern of cultural eutrophication on page 133 without the spurious oscillations.

A Stella Model to Match Anderson's Dynamo Model

Figure 1 shows a Stella model to implement Anderson's Dynamo model shown in Figure 5-4. The initial inflow of nutrients is set at 0.01 mg/liter and the growth is set at 0.02/year. The initial value of the carbon in the nutrient pool is 36 mg/liter, and the growth rate is set at 0.2/year. The initial value of the biomass is 0.425 mg/liter and the biomass is subject to a respiration rate of 0.445/year and a death rate of 4/year. The initial value of the detritus is 5 mg/liter, and the decay rate is 0.0445 /year. The total carbon in the lake at the start of the simulation is 41 mg/liter, and this total will grow over time as nutrients enter the lake. (Notice that nutrients never leave the lake.)

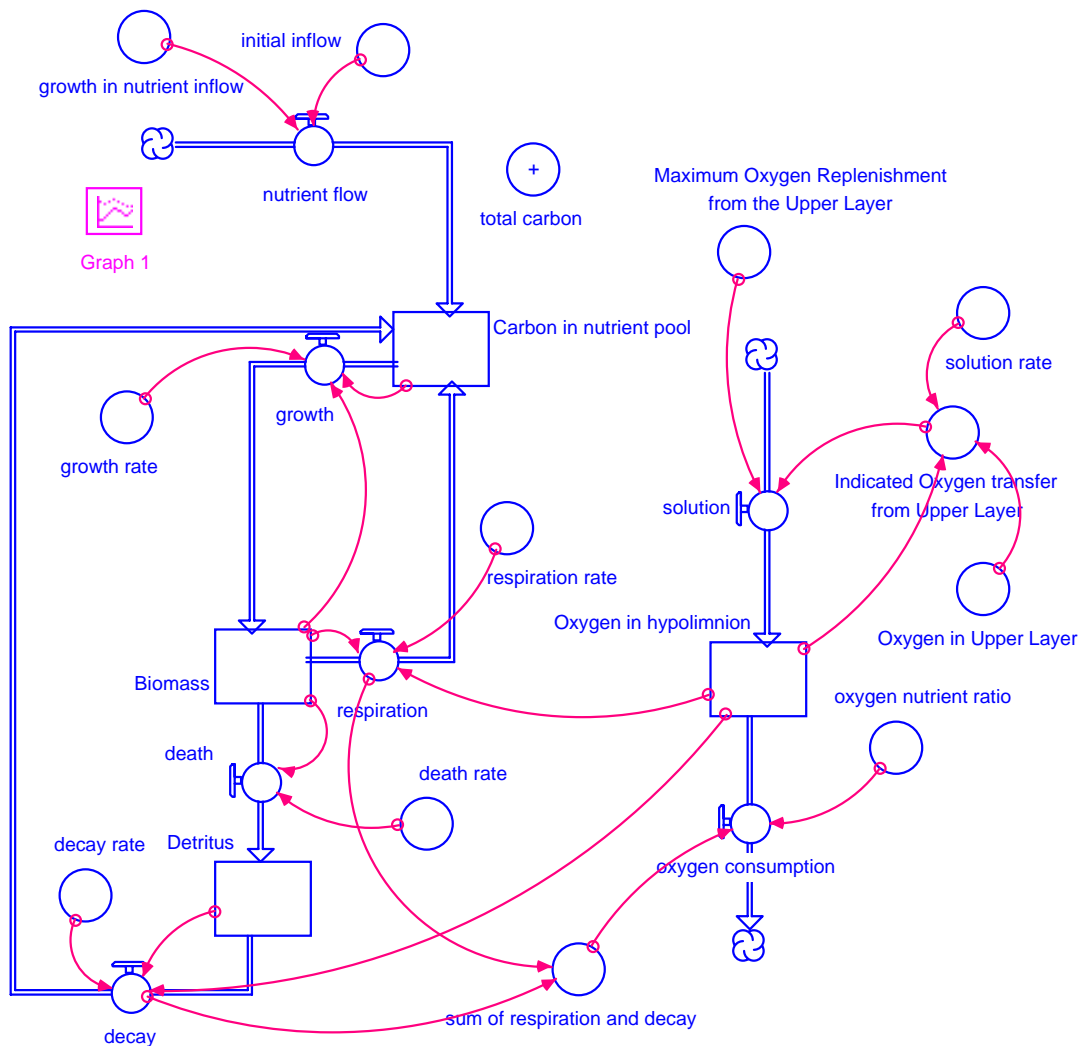


Figure 1. Stella stock-and-flow Diagram to Match Anderson's Model on page 129.

The right side of Figure 1 shows the oxygen in the hypolimnion as a stock variable. This is initially 7 mg/liter and represents the concentration of oxygen in the lower layer of the lake. The upper layer concentration is set at 9.5 mg/liter. Oxygen is transferred to the lower layer by the "solution" flow which is set at an indicated value or

the maximum value, whichever is smaller. The maximum annual replenishment is estimated at 20 mg/liter per year. (Anderson compresses a lot into the equation for the “solution” flow, so I have added an “indicated oxygen transfer from upper layer” for clarity.) The indicated transfer depends on the gap between the upper layer and lower layer concentrations. If the two concentrations are the equal, there is no transfer. If the two concentrations differ by 1 mg/liter, Anderson assumes the indicated transfer is 4.27 mg/liter. That is, his “solution rate” is 4.27/year. (This is an unusually high transfer rate in a model that will run for 200 years, and it is one of the factors contributing to the need for a very small DT in the model.) Anderson states that the mechanisms for oxygen transfer are not “not well understood” (see top of page 128), so he estimates the solution rate at 427%/year based on “steady state conditions” (see bottom of page 128).

Oxygen is removed from the lower layer due to respiration of the biomass and decay of the detritus. (I have added these two flows together for clarity.) The sum of these activities is multiplied by 2.67, the “oxygen nutrient ratio.” This ratio is based on the fact that “one gram of carbon is stoichiometrically equivalent to 2.67 grams of oxygen” (bottom of page 124).

If you build the Stella model in Figure 1 and simulate it for 200 years with the DT at 1/32 of a year, you should get the results shown in Figure 2. The biomass is scaled from 0 to 2 mg/liter; the oxygen from 0 to 10 mg/liter, and the remaining variables from 0 to 50 mg/liter. Figure 2 shows the pattern of cultural eutrophication for the first 150 years. Then we see spurious oscillations in the final years of the simulation.

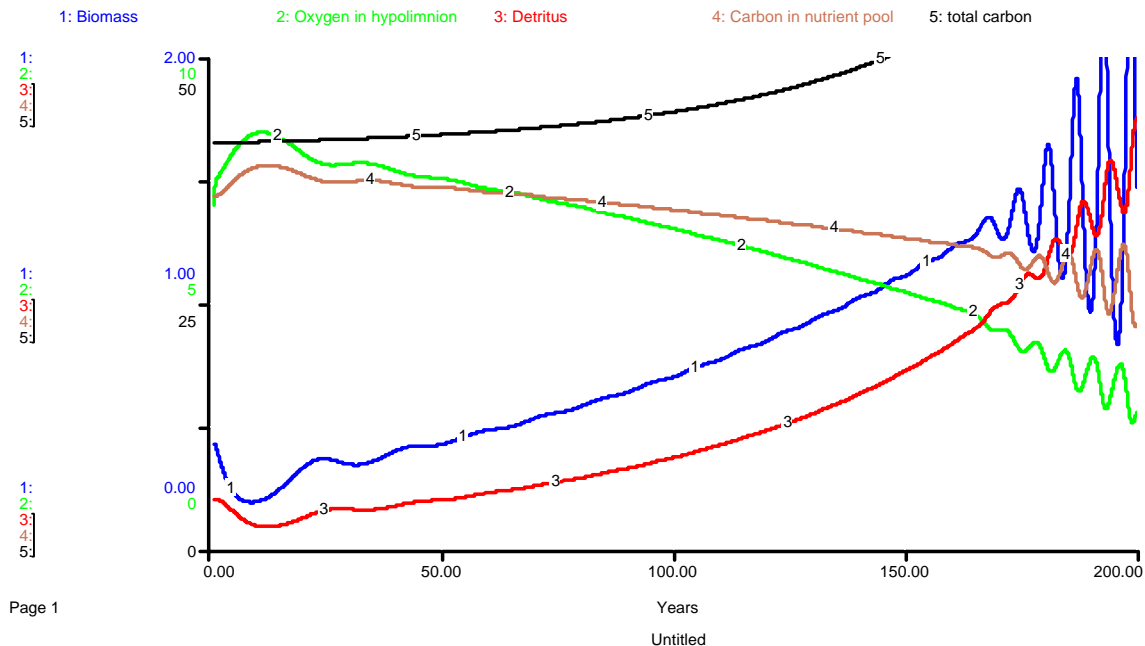


Figure 2. Reproducing Anderson’s Results from Page 133.

Simulating Oxygen in the Lower Layer in a Separate Model

Figure 3 shows a separate model to study the adjustment in the oxygen concentration in the lower layer if the sum of respiration and decay is known. The sum of respiration and decay is a user-specified input which we will increase over time. The model implements all of Anderson's assumptions for oxygen in the lower layer. For example, the solution rate is set at 427%/year and the maximum annual replenishment from the upper layer is set at 20 mg/liter per year. The oxygen nutrient ratio is set at 2.67 and the oxygen in the upper layer is set at 9.5 mg/liter.

The purpose of this model is to learn the response of the oxygen to different levels of stress caused by the sum of respiration and decay. (What I am doing here is somewhat similar to the population model changes on the top of page 118 in Modeling the Environment.) I hope to learn how the oxygen will respond. If I see a clear pattern of response, I will put the "pattern" into a new model of eutrophication.

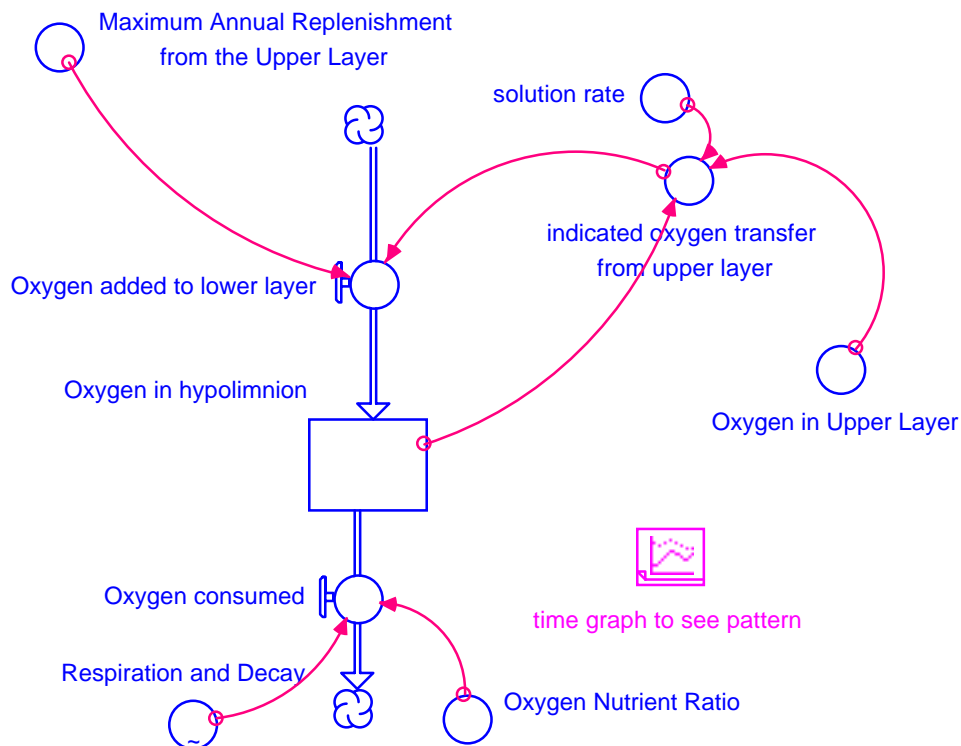


Figure 3. Separate model of Oxygen in the Hypolimnion.

Figure 4 shows the time graph results that will allow us to learn the pattern expected from Anderson’s assumptions. The simulation begins with respiration and decay at zero, so the oxygen in the lower layer is at 9.5 mg/liter (the same as the upper layer). When we stress the model with respiration and decay set at 2 mg/liter, it responds by lowering the oxygen to 8.2 mg/liter. If the stress is increased to 4 mg/liter, the oxygen declines to 7.0 mg/liter. When the stress reaches 12 mg/liter, the oxygen in the lower layer will be down to 2 mg/liter.

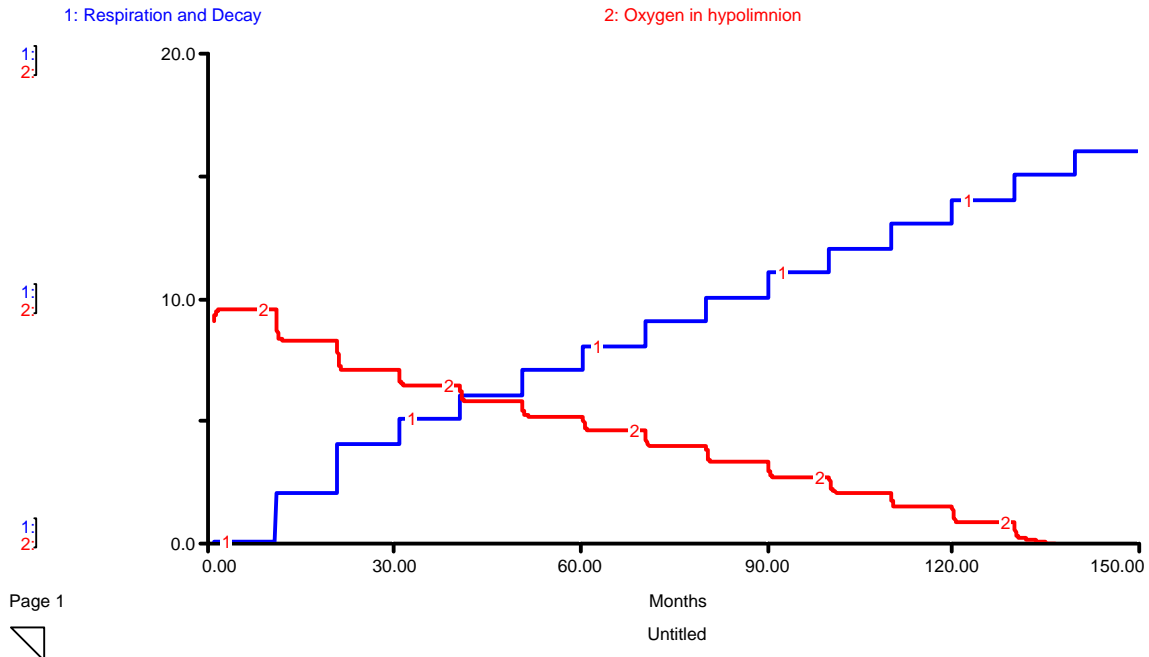


Figure 4. Response of the Model in Figure 3 to a Wide Range of Values for Respiration and Decay

A New Stella Model of Eutrophication

Figure 5 shows a new model which takes advantage of what we see in Figure 4. The stock of oxygen in the lower layer is still a stock variable, but this stock is not subjected to flows that operate on extremely small time constants (such as a solution rate of 427%/year). Rather, the stock is controlled by a bi-flow which will move the stock of oxygen in the lower layer up or down depending on the “Oxygen in the Lower Layer Expected from the Separate Model.” A ~ for this variable reminds us that a graph function is used to tell us the expected value of the oxygen in the lower layer. We set the entries in this graph based on the results shown in Figure 4. If the sum of respiration and decay were 2 mg/liter, for example, we set the expected value of oxygen in the lower layer at 8.2 mg/liter. If the sum were 4 mg/liter, the expected value of oxygen in the lower layer would be 7.0 mg/liter.

Now, how fast should the bi-flow adjust the oxygen in the lower layer to the expected value? We control the pace of adjustment with the “time interval for oxygen to adjust.” Anderson says the mechanisms are not well understood, but he seems to be thinking in terms of “seasonal turnover” for replenishing the oxygen in the lower layer. For this exercise, I set the “time interval for oxygen to adjust” to 1 year.

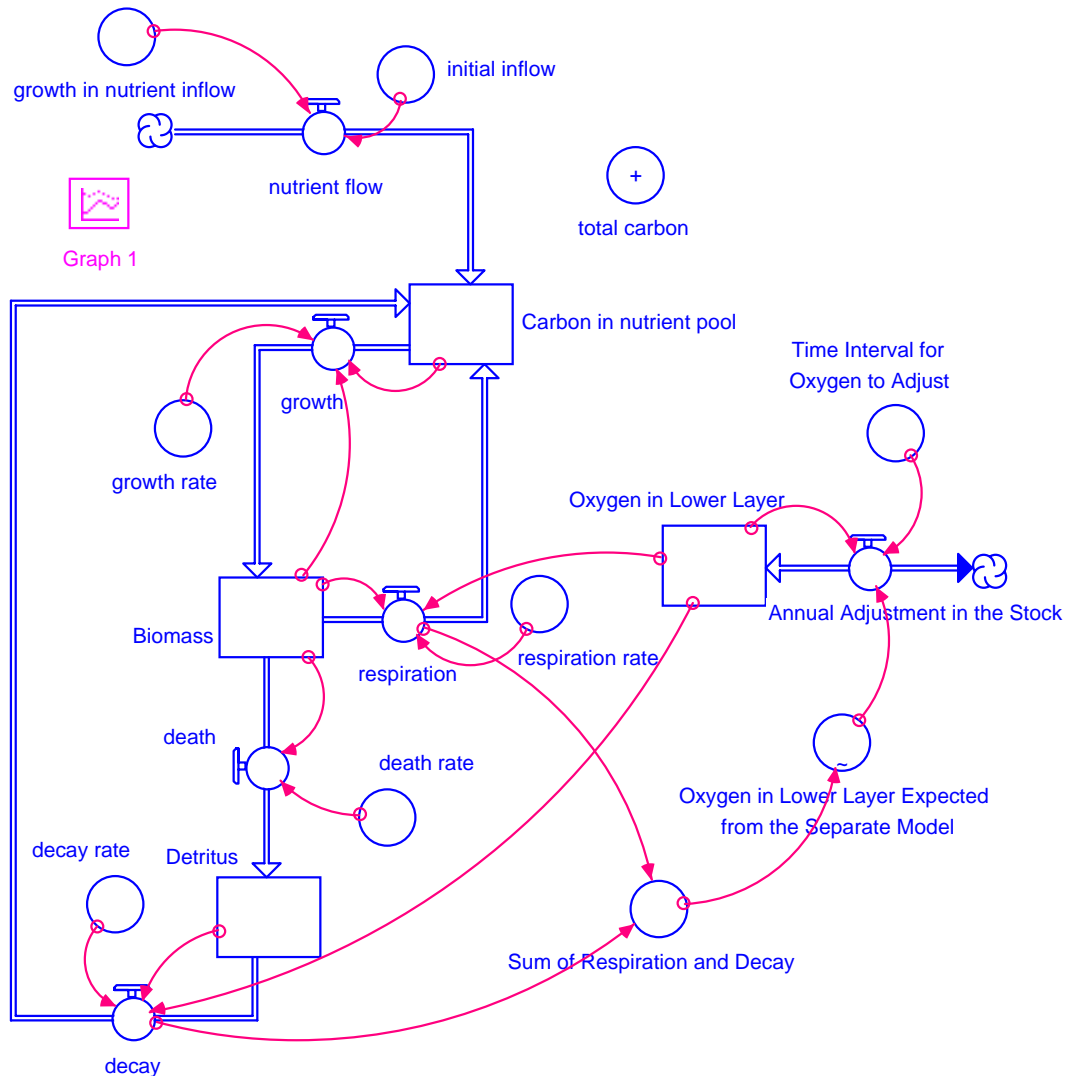


Figure 5. A Different Approach to Including Oxygen in the Lower Layer within Anderson’s Model of Eutrophication.

Notice that the stock of oxygen in the lower layer is connected to the rest of the model in the same manner as before. That is, the oxygen concentration in the lower layer acts (multiplicatively) to control the flow “respiration” and the flow “decay.” The left side of Figure 5 is set up with the same parameters as before. If you simulate this model for 200 years with DT set at 1/32 year, you should see the results in Figure 6.

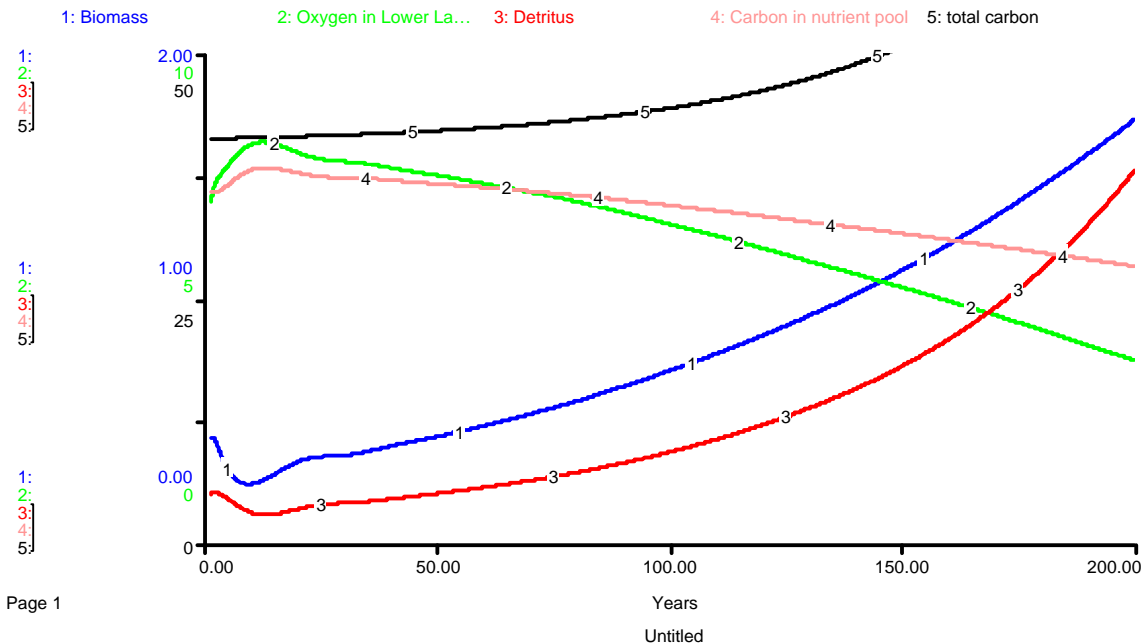


Figure 6. Results When the Model in Figure 5 is Used to Simulate Cultural Eutrophication.

Figure 6 shows the desired result. We see the general pattern of cultural eutrophication that is similar to the first 150 years of results on page 133 of Toward Global Equilibrium. By the 150th year, for example, the biomass has grown by around 3 fold and the detritus by around 4 fold. The oxygen in the lower layer had declined by over a third.

The important result from Figure 6 is that the pattern of cultural eutrophication continues past the 150th year without the spurious oscillations. As more and more nutrients enter (and never leave) the lake, the build up of biomass and detritus continues. The depletion of oxygen continues as well.

Conclusion

I believe the model in Figure 5 is a more useful tool for simulating cultural eutrophication than the model shown in Figure 1. Students working on the advanced exercises might make use of the new model.

If you do use the new model, don't forget that it was developed by first examining the oxygen response with the separate model shown in Figure 3. If you simulate a new system (i.e., with a different oxygen nutrient ratio), you would need to build the model in Figure 3 and repeat the pattern testing shown in Figure 4.