

HANDOUT I.2: Sex linkage and Hardy-Weinberg

Let's consider a diploid population with X-Y sex determination (females are XX ; males are XY). We want to study evolution of a locus with two alleles on the X-chromosome (with no counterpart on the Y-chromosome).

Some notation:

$p_f(t)$ = frequency of the A allele among X gametes in females in generation t ;

$p_m(t)$ = frequency of the A allele among X gametes in males in generation t .

Under the H-W assumptions, the following offspring genotype frequencies are found:

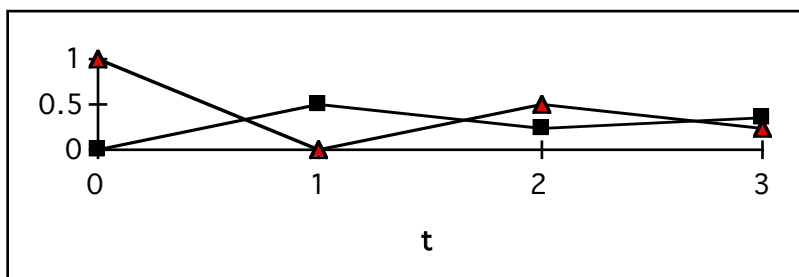
Daughters	Sons
$AA: P_{AA}(t+1) = p_f(t) p_m(t)$	$AY: P_{AY}(t+1) = p_f(t)$
$Aa: P_{Aa}(t+1) = p_f(t)[1 - p_m(t)] + p_m(t)[1 - p_f(t)]$	$aY: P_{aY}(t+1) = 1 - p_f(t)$
$aa: P_{aa}(t+1) = [1 - p_f(t)][1 - p_m(t)]$	

Allele frequencies among the offspring (computed from these genotype frequencies) are:

- $p_f(t+1) = \frac{1}{2} [p_f(t) + p_m(t)]$ (average of allele frequencies in both parent sexes)
- $p_m(t+1) = p_f(t)$ (the allele frequency among just the *female* parents)

Evolutionary Dynamics: Suppose we have the extreme case $p_m(0) = 1, p_f(0) = 0$:

t	0	1	2	3
$p_m(t)$	1	0	0.5	0.25
$p_f(t)$	0	0.5	0.25	0.375



Unlike cases we've seen up until now, the evolutionary paths oscillate towards an equilibrium. What equilibrium is eventually reached? It turns out that the frequency of the A allele becomes $p_{eq} = \frac{1}{3} p_m(0) + \frac{2}{3} p_f(0)$ in both sexes. In the above case, $p_{eq} = \frac{1}{3}(1) + \frac{2}{3}(0) = \frac{1}{3}$.