SELECTION & THE SINGLE DIALLELIC DIPLOID LOCUS. 1

• General equations

– Diploid selection to asexual/haploid selection except that parental <u>genotypes</u> not passed on, although their <u>alleles</u> are.

– Will repeatedly use the following facts from our H-W discussion :
  (1) Offspring <u>allele</u> frequencies = those of their randomly mating parents
  (2) Offspring <u>genotype</u> frequencies are in H-W proportions, even if their parents' were not.

– **Relative fitnesses** of diploid genotypes: w_{AA} , w_{Aa}, w_{aa}

– frequency of A allele = p; frequency of allele $a = q = 1 - p$.
  • assume zygotes are in H-W proportions.

– Life Cycle: $\underset{\text{gen. } t}{\text{zygotes}} \xrightarrow{\text{survival}} \underset{*}{\text{adults}} \xrightarrow{\text{fecundity}} \text{gametes} \xrightarrow{\text{random union}} \underset{\text{gen. } t+1(')}{\text{zygotes}}$

– Assume all have same fecundity so relative fitness = viability

– <u>Genotype</u> <u>frequencies</u> after selection (but before reproduction)

  • denoted by *

  • $$P^*_{AA} = \frac{\text{number of } AA \text{ parents surviving}}{\text{total number of surviving parents}} = \frac{Np^2 w_{AA}}{Np^2 w_{AA} + N2pqw_{Aa} + Nq^2 w_{aa}} = p^2 \frac{w_{AA}}{\overline{w}}$$

  where $\overline{w} = p^2 w_{AA} + 2pqw_{Aa} + q^2 w_{aa}$ is the population **mean fitness**.

  • Similarly, $P^*_{Aa} = 2pq\dfrac{w_{Aa}}{\overline{w}}$, $P^*_{aa} = q^2 \dfrac{w_{aa}}{\overline{w}}$.

– <u>Allele</u> <u>frequencies</u> after selection and reproduction

  • If surviving parents mate randomly, then
  $$p' = p^* = P^*_{AA} + \tfrac{1}{2} P^*_{Aa} = p^2 \frac{w_{AA}}{\overline{w}} + pq\frac{w_{Aa}}{\overline{w}} = p\frac{\left(pw_{AA} + qw_{Aa}\right)}{\overline{w}}$$

  • Quantity in parentheses is **mean fitness of individuals who carry *A***:
  $$\overline{w}_A \equiv pw_{AA} + qw_{Aa}$$

• Can then rewrite above as, $p' = p\frac{\overline{w}_A}{\overline{w}}$.

• Similarly, $q' = q\frac{\overline{w}_a}{\overline{w}}$ where $\overline{w}_a \equiv pw_{Aa} + qw_{aa}$ is the **mean fitness of individuals who carry *a*.**

– <u>Handy</u> <u>Fact</u>: $\overline{w} = p\overline{w}_A + q\overline{w}_a$ (compare with *asexual* mean fitness)

– Rate of allele frequency change ($\Delta p = p' - p$): $\Delta p = pq\frac{\overline{w}_A - \overline{w}_a}{\overline{w}}$

– <u>Summarize</u>:

• Equations for evolution by selection in diploids identical (in form) to asexuals/haploids.

• Allele frequency change (evolution) depends on **genetic variation** and **fitness differences**

• Spread of an advantageous allele

– Designate A to be the advantageous allele.

– Adopt the following notation:

Genotype	AA	Aa	aa
Fitness	$w_{AA} = 1$	$w_{Aa} = 1 - hs$	$w_{aa} = 1 - s$

– h measures allelic **dominance** of A over a:
• $h = 1 \leftrightarrow 1 : 1 - s : 1 - s$ (A is <u>recessive</u> advantageous)
• $h = 0 \leftrightarrow 1 : 1 : 1 - s$ (A is <u>dominant</u> advantageous)
• $h = 1/2 \leftrightarrow 1 : 1 - s/2 : 1 - s$ (semi-dominant, additive, no dominance)
• $h \neq 0, 1, \frac{1}{2}$ (partially dominant)

– Equations in this parameterization for allele frequency evolution:

$$\Delta p = pq\frac{s[q + h(p - q)]}{1 - sq(q + 2ph)}$$

– Application (from Y.J. Chung. Genetics. 1967)

- Selection against allele that produces stubble bristles in *Drosophila*

Genotype	+/+	+/sb	sb/sb
Phenotype (Fitness)	normal	stubble	lethal in larval stages ($s = 1$)

– <u>Two</u> <u>Special</u> <u>Cases</u>:

(1) <u>Dominant</u> advantageous allele $\left(h = 0,\ \begin{matrix} w_{AA} \\ 1 \end{matrix} : \begin{matrix} w_{Aa} \\ 1 \end{matrix} : \begin{matrix} w_{aa} \\ 1-s \end{matrix}\right)$

$$\Delta p = p(1-p)\frac{s(1-p)}{1-s(1-p)^2}$$

- Dynamics: spread of allele is rapid at first; later, slows as A spreads.
- Why these dynamics?
  - – Biological Intuition
  - – Mathematical Explanation

(2) <u>Recessive</u> advantageous allele $\left(h = 1,\ \begin{matrix} w_{AA} \\ 1 \end{matrix} : \begin{matrix} w_{Aa} \\ 1-s \end{matrix} : \begin{matrix} w_{aa} \\ 1-s \end{matrix}\right)$

$$\Delta p = p(1-p)\frac{sp}{1-s(1-p^2)}$$

- Dynamics: spread of allele A is slow at first; rapid later as A spreads.
- Why these dynamics?
  - – Biological Intuition
  - – Mathematical Explanation

– <u>Rates</u> <u>of</u> <u>evolution</u>: Dominant vs. Recessive Alleles

- Basic Principles:
  – rare alleles occur primarily in heterozygotes

- intuition for the magnitude of change when an allele is rare: compare heterozygous fitness to that of the common homozygote.

• Main Features:

- Extensive time needed for spread of advantageous recessive (extensive time needed for elimination of deleterious recessive)

- Rapid evolution from 0.1 to 0.9 in all cases

- Multiplicative selection: produces even faster evolution (homework)

• For weak selection ($s << 1$), time required for a specified allele frequency change is inversely proportional to s, i.e., $t \propto 1/s$ when $s << 1$.