**MATH 273: Practice Exam 1**

**1.** (a) Determine the arc length function $s(t)$ for the vector-valued function $\mathbf{r}(t) = \left\langle t, \ln t^2, \dfrac{2-t}{t} \right\rangle$, $t \geq 1$.

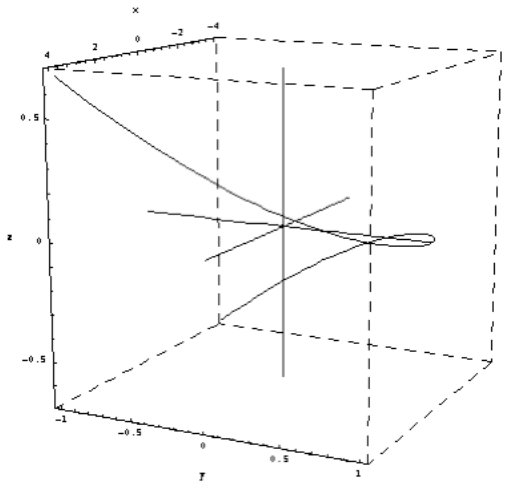
(b) Calculate the length of the arc from $t = 1$ to $t = 4$ for the vector function $\mathbf{r}$ in part (a).

**2.** The graph of the vector-valued function $\mathbf{r}(t) = \left\langle 4t, 1 - 2t^2, \frac{2}{3}t^3 \right\rangle$ is shown below.

(a) Draw an arrow on the graph to indicate the direction of $\mathbf{r}(t)$ with increasing $t$.

(b) Derive parametric equations for the tangent line to the graph of $\mathbf{r}(t)$ at $t = 0$. Sketch and label the line on the figure below.

(c) Compute $\mathbf{T}(0)$ and $\mathbf{N}(0)$. Sketch and label these vectors on the figure below.



**3.** Let $f(x,y)=\dfrac{y^2\sin x}{x^2+y^2}$.

(a) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the $x$-axis.

(b) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the $y$-axis.

(c) Find $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ along the line $y=mx$.

(d) Based on your results in parts (a)–(c), what do you conclude about $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

**4.** Let $f(x,y)=\dfrac{\sin(xy)}{xy}$. It can be shown that $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ exists for all $(x_0,y_0)\in\mathbf{R}^2$. Is $f(x,y)$ continuous? Explain.

**5.** Find all the second partial derivatives of the function $f(x,y)=\sqrt{x^2+y}$.