MATH 273: Practice Exam Questions 2

1. Find parametric equations of the normal line to the surface described by the equation $z^2 - 4y^2 - x^2 = 0$ at (3, 2, –5).

2. For the function $f(x, y) = x^2\left(y^3 - \frac{1}{4}\right)$ you are given that $\nabla f(x, y) = \left\langle 2xy^3 - \frac{1}{2}x, 3x^2y^2 \right\rangle$ and $\nabla f(2, -1) = \langle -5, 12 \rangle$.

(a) Compute the directional derivative of $f(x, y)$ in the direction $-3\hat{i} - 4\hat{j}$ at the point (2, –1).

(b) Find the maximal value of the directional derivative of $f(x, y)$ at the point (2, –1).

(c) Give a tangent vector to the level curve of $f(x, y)$ at the point (2, –1).

3. Find an approximate value for $f(0.9, 3.1)$ using only the following information about $f(x, y)$ and its derivatives: $f(1,3) = -1$, $f_x(x,y) = y/x$, and $f_y(x,y) = \left[f(x, y)\right]^2$.

4. Find and classify all local maxima, local minima, and saddle points of $f(x, y) = -y^4 + 4xy - 2x^2 - 1$.

5. Assume the equation $z \ln y + x \ln z = xy$ defines z as a differentiable function of x and y. Find $\partial z / \partial x$.

6. Let $z = y^2 - xy + x^2$, $x = \sin t$, and $y = \cos(2t)$.

(a) Give the proper notation and chain rule expression for the derivative of z with respect to t.

(b) Find this rate of change when $t = \pi/4$.