

### MATH 273: Practice Final Exam Questions

- Let  $\hat{r}(t) = \langle 3\sin t, 2 + 4\sin t, 3 + 5\cos t \rangle$ . Find the unit tangent and unit normal vectors to the curve swept out by  $\hat{r}(t)$  at the point  $t = \frac{\pi}{2}$ .
- Suppose  $z = f(x, y)$ ,  $x = g(u, v)$ , and  $y = h(u, v)$ . Give the proper notation and chain rule expression for the rate of change of  $z$  with respect to  $v$ . **Do not calculate any derivatives at this point.**
  - Calculate the derivative in part (a) assuming  $z = x \cos(xy)$ ,  $x = ue^v$ , and  $y = v \ln u$ .
  - Use your answer to (b) to find the rate of change of  $z$  with respect to  $v$  when  $u = 1$  and  $v = 0$ .
- You are given the function  $f(x, y) = y^2 + e^{(\sin x)y}$ , its gradient  $\nabla f(x, y) = \left\langle \frac{\cos x}{y} e^{(\sin x)y}, 2y - \frac{\sin x}{y^2} e^{(\sin x)y} \right\rangle$ , and the evaluation  $\nabla f(0, -1) = \langle -1, -2 \rangle$ .
  - What is the maximum rate of increase of  $f(x, y)$  at the point  $(0, -1)$ ?
  - Determine the unit vector in the direction of greatest decrease of  $f(x, y)$  at the point  $(0, -1)$ .
  - Give a unit vector tangent to the level curve of  $f(x, y)$  at the point  $(0, -1)$ .
- Find and classify all local maxima, local minima, and saddle points of  $f(x, y) = x^2 - 6xy + 2y^3 - 8x - 16$ .
- Evaluate  $\int_0^2 \int_{\sqrt{x}}^3 (x + y^3) dy dx$
  - Give an equivalent iterated integral with the order of integration reversed. **DO NOT EVALUATE THIS INTEGRAL**

6. Let  $R$  be the region in the first quadrant of the  $xy$ -plane bounded between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 9$ .
- (a) Set up an appropriate iterated integral in rectangular coordinates equivalent to  $\iint_R f(x, y) dA$ .
- (b) Set up an appropriate iterated integral in polar coordinates equivalent to  $\iint_R f(x, y) dA$ .
7. Consider the region  $E$  which is bounded by the paraboloid  $z = 4x^2 + 4y^2$  and the plane  $z = 4$  (see figure). Set up a triple integral in rectangular coordinates to find the volume,  $V = \iiint_E dV$  of  $E$ . **DO NOT EVALUATE.**
8. Compute the line integral  $\int_C (x^2 + y) ds$  where  $C$  consists of the line segment from  $(2, 0)$  to  $(-2, 0)$  and the lower half of the circle  $x^2 + y^2 = 4$ .
9. (a) Show that  $\hat{F}(x, y) = \langle 5x^4 - 8xy^3, -12x^2y^2 \rangle$  is a gradient field of  $f(x, y) = x^5 - 4x^2y^3 + 3$ .  
(b) Find the work done by the force  $\hat{F}(x, y) = \langle 5x^4 - 8xy^3, -12x^2y^2 \rangle$  acting on a particle moving along the curve  $C$  parameterized by  $\hat{r}(t) = \langle t^3, t \rangle$  as  $t$  goes from 0 to 1.