Key to Problem Set #1

1. (a) $rank(\mathbf{A}) = 2$.

   (b) $\begin{pmatrix} \underline{1} & 0 & -2 & 1 \\ 0 & \underline{1} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ (Underlined entries are pivots.)

   (c) Basic columns: $\mathbf{A}_{*1}, \mathbf{A}_{*2}$. Nonbasic columns: $\mathbf{A}_{*3}, \mathbf{A}_{*4}$.
   Relationships: $\mathbf{A}_{*3} = -2\mathbf{A}_{*1} + \mathbf{A}_{*2}$, $\mathbf{A}_{*4} = \mathbf{A}_{*1}$

2. (a) Gaussian elimination will not introduce nonzero elements into a column containing only zeros. Consequently, a column containing all zeros cannot contain a pivot since pivots, by definition, must be nonzero. Thus, a column with all zeros cannot be basic.

   (b) Consider the matrix
   $$\begin{pmatrix} \underline{*} & * & * \\ 0 & \underline{*} & * \end{pmatrix}$$
   where each * represents a nonzero entry. This matrix is in row echelon form; the pivots are underlined. Although the 3rd column contains no zero entries, it also has no pivot and so is nonbasic.

3. The system is consistent for any values of b_1 and b_2.

4. **A** is symmetric but not skew-symmetric, hermitian, or skew-hermitian.

5. Conjugate transposition is not linear since $(\alpha\mathbf{A})^* = \bar{\alpha}\mathbf{A}^* \neq \alpha\mathbf{A}^*$.