Exercise Set #2

1. Gillespie Problem 3.4

2. Gillespie Problem 3.5 (computer program not required)

Do at least one of the following two exercises

3. Consider the "additive" fitnesses $w_{11} = 1$, $w_{12} = 1 - s/2$, $w_{22} = 1 - s$. Let p = freq(A_1), q = freq(A_2); prime (') indicates "next generation."

   a. Let $\Delta p = p' - p$ and $\Delta q = q' - q$. Use the fact that $q = 1 - p$ to show $\Delta q = -\Delta p$

   b. Use equation 3.1 or 3.2 of Gillespie (either edition) to show that
   $$\Delta p = \frac{pqs}{2\overline{w}}$$

   c. Show that the mean fitness in this case is $\overline{w} = 1 - qs$

   d. Use a–c to show that
   $$\Delta\overline{w} = \overline{w}' - \overline{w} = \frac{pqs^2}{2\overline{w}}$$

   e. The variance of random variable X is $var(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2$ where $E(\cdot)$ means expectation or average of the argument over the distribution of X. Thinking of the additive fitnesses as random variables sampled from a population in Hardy-Weinberg proportions, show that the variance in fitness is
   $$var(w) = \frac{pqs^2}{2},$$
   which, combined with part d, implies that
   $$\Delta\overline{w} = \frac{var(w)}{\overline{w}}.$$
   This is a version of "Fisher's fundamental theorem of natural selection" for the case of additive fitnesses.

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4. Write a program that iterates directional selection for a diploid population with two alleles, A_1 and A_2, using the deterministic recursion for allele frequency change (eq. 3.1 or eq. 3.2 in either edition of Gillespie). Assume A_1 is favored and fitnesses are "additive," i.e., that $w_{11} = 1$, $w_{12} = 1 - (s/2)$, and $w_{22} = 1 - s$, where $s > 0$. Your program should track the evolution of p, the frequency of A_1. Use

your program to compute the minimum number of generations needed to evolve from p_0 to p_1 for each of the following cases:

Case	p_0	p_1	s
A	0.01	0.99	0.001
B	0.01	0.99	0.01
C	0.01	0.99	0.1
D	0.001	0.99	0.001
E	0.001	0.99	0.01
F	0.001	0.99	0.1
G	0.01	0.999	0.1
H	0.001	0.999	0.1

(b) Use your computational results to conjecture some general hypotheses or rules-of-thumb about how the starting and final frequencies and the strength of selection affect the time required for an allele spread. Be sure to explain your reasoning.