## Mathematical Genetics, Fall 2017, HW 5

Instructions: Do all problems and show your work.

1. Consider the following table of SNP data (with sequence labels on the left and site labels across the top):

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{a}$ | G | C | G | T | T | A |
| $\mathbf{b}$ | G | C | G | T | T | A |
| $\mathbf{c}$ | C | C | G | T | T | T |
| $\mathbf{d}$ | G | G | C | A | A | T |
| $\mathbf{e}$ | G | G | C | T | T | T |

Assume that the ancestral sequence is GGGTTT and that we have infinite sites mutation.
(a) Compute Watterson's estimate of $\theta$.
(b) Compute Tajima's estimate of $\theta$.
(c) Compute the (unfolded) site frequency spectrum.
(d) Compute the numerators for Tajima's $D$ statistic and Fu and Li's $D^{*}$.
(e) Draw a (properly labeled) coalescent tree that is consistent with this data set.
2. Consider a WF model with fast fluctuations in population size, where there are 3 sizes: $N_{1}=N, N_{2}=4 N, N_{3}=N / 2$, and the proportions of time spent at these sizes is $0.2,0.3$, and 0.5 , respectively.
(a) In the Kingman coalescent obtained by letting $N \rightarrow \infty$, what is the pairwise coalescence rate?
(b) What is the mean time until the first coalescence in a sample of size 10 ?

