CHEMISTRY
Error Analysis for Laboratory Reports

There are ***three steps*** in error analysis of most experiments.

1. *Propagation of errors* can be performed even before the experiment is performed.
2. *Measuring the errors* is done during the experiment.
3. *Comparison with accepted values* is performed after the experiment is completed.

This means a **numerical** error analysis, ***not*** a post-experimental accounting of what you could do to improve your results.
Available source of information on error analysis is ***in Chapter 2 of your laboratory textbook***.

Your laboratory report needs to include the following:

- ✓ For three or more point repetitions, do the average and standard deviation for each point (see chapter 2 of the textbook or a quantitative analysis text).
- ✓ Propagate measurement errors through calculations.
- ✓ Report all calculated values with error and units attached, e.g. 4.2 ± 0.1 mL.
- ✓ Compare results with accepted and/or literature values

Example of Propagation of Errors

One of the experiments involves measuring the heat capacity ratio for a certain gas, say Argon. Two equations are used in this experiment:

$c = \upsilon\lambda$ and $\gamma = \frac{\mathrm{Mc}^2}{\mathrm{RT}}$ where γ is the heat capacity ratio, $\frac{\mathrm{C_p}}{\mathrm{C_v}}$.

I. The variables are υ, λ, and T.

1. Estimate the error in the measurement of these three quantities. To do this you must decide how accurate the measurement was to within a certain number of units and you must be realistic.

2. Average values ± uncertainties

$\upsilon = 2207 \pm 2\text{Hz}$,
$\lambda = 144.8 \pm 0.5$ mm
$T = 296.8 \pm 0.2$ K.

Since we must use the measured values to find c first, we must first propagate through that equation to find the error in the calculated value of *c*.
Use equation 52 on page 58 of the text. In this case, $F(x,y) = F(\upsilon,\lambda) = c = \upsilon\lambda$.

$$\Delta^2 F = \left[\left(\frac{\partial F}{\partial x}\right)^2 \Delta^2 x + \left(\frac{\partial F}{\partial y}\right)^2 \Delta^2 y\right]$$

Substituting *c* in for F gives an equation of the form:

$$\Delta^2 c = \left[\left(\frac{\partial c}{\partial \upsilon}\right)^2 \Delta^2 \upsilon + \left(\frac{\partial c}{\partial \lambda}\right)^2 \Delta^2 \lambda\right]$$

After the actual derivations:

$$\Delta^2 c = \lambda^2 \Delta^2 \upsilon + \upsilon^2 \Delta^2 \lambda$$

Plugging the numbers from above into this equation gives

$$\Delta^2 c = (144.8)^2 (2)^2 + (2207)^2 (0.5)^2 = 83{,}868.2 + 1{,}217{,}712.2 = 1{,}301{,}580.4$$

and

$$\boxed{\Delta c = 1140.8}$$

The result should be written as **c = 319.5 ± 1.1 m/sec**

II. Value of γ and its propagated error.

$$\gamma = \frac{Mc^2}{RT} = \frac{0.039948(319.5)^2}{8.324(296.8)} = 1.65$$

M_{Ar} = 39.948 g/mol = 0.039948 Kg mol^{-1}
Gas constant R = 8.314 J mol^{-1} K^{-1}
T = 296.8 ± 0.2 K and c = 319.5 ± 1.1 m/sec

The propagation equation will look like:

$$\Delta^2 \gamma = \left[\left(\frac{\partial \gamma}{\partial c}\right)^2 \Delta^2 c + \left(\frac{\partial \gamma}{\partial T}\right)^2 \Delta^2 T = \left(\frac{2Mc}{RT}\right)^2 \Delta^2 c + \left(\frac{-1Mc^2}{RT^2}\right)^2 \Delta^2 T\right]$$

Again plugging in the values from above into the equation $\Delta^2\gamma$ becomes:

$$\Delta^2 \gamma = \left(\frac{2x(0.039948)x319.5}{8.314x(296.8)}\right)^2 (1.1)^2 + \left(\frac{-0.039948x(319.5)^2}{8.314x(296.8)^2}\right)^2 (0.2)^2 = 1.29x10^{-4} + 1.24x10^{-6}$$

$\Delta\gamma = 0.01$ and then the value for the heat capacity ratio with its error is then

$$\boxed{\boldsymbol{\gamma = 1.65 \pm 0.01}}$$

Note that from the above propagation of error one could point out which quantity contributes the most to the error. The first term ($1.29x10^{-4}$) is 2 orders of magnitude larger than the second term. By improving the way *c* is measured the error on γ could be reduced.