

Scaling Relationships

Most single-celled organisms are microscopic; only the largest of them are visible to the naked eye. The cells that make up multicellular organisms are also microscopically small. Very large cells are rare and are made for special circumstances (such as an egg of an ostrich). So why are most cells small? The answer lies in how cells interact with their environment. Everything that enters or leaves a cell must do so through the cell surface. Typically, as the volume of a cell increases, the surface area per unit volume decreases. A large cell simply could not bring in materials and get rid of waste fast enough because of its limited surface area relative to its volume. Some multi-cellular organisms, such as flat worms, absorb nutrients directly through their entire body surface. Their flattened shape maximizes the surface area to volume relationship. Larger, more complex animals must rely on adaptations such as circulatory systems and specialized organs for gas exchange, nutrient absorption, and waste removal.

In the first exercise, we will measure diffusion into different sizes of “cells” represented by blocks of agar. The agar has been dyed with a pH indicator that is red at neutral pH but loses that color in acidic conditions.

Exercise 1a - Surface Area to Volume Relationships, Diffusion

Materials

- . • Agar blocks made with phenolphthalein indicator
- . • Ruler
- . • Knife
- . • Vinegar
- . • Paper towels
- . • Soaking pan
- . • Graph paper

Procedure

1. Invert the container with the colored agar to release the block of agar onto a paper towel. Trim the block so that the sides are straight up and down rather than sloping. Measure and cut blocks with the following dimensions: 1) 1x1x1 cm, 2) 2x2x2 cm; 3) 3x3x3 cm, 4) 4x4x4 cm, 5) 1x2x2 cm. We suggest that you cut the largest block first. You should have plenty of agar for all the blocks. Return unused pink agar into a bucket in the front of the class.
2. Arrange the cut blocks in the container so that they do not overlap. Pour in enough vinegar to cover the blocks completely.
3. Allow the blocks to soak in the acidic vinegar just until the smallest cube loses all of its pink color. While you wait for this to happen, do Exercise 1b.

4. When the smallest cube has lost all of its pink color, immediately remove all of the blocks from the vinegar and put them on a paper towel.
5. Measure the dimensions of the pink area within all blocks that still have a pink center. Calculate the volume of the pink area in each block (Volume = length * width * height). Enter these volumes in the table below.
6. Measure the distance that the vinegar diffused from each edge. Average the distance for each block and enter it in the table below.
7. Calculate the volume into which the vinegar diffused for each block (diffused volume = total volume – pink volume). Enter this volume into the table below.
8. Calculate the percent of the total volume into which the vinegar diffused and enter it in the table below. (Percent diffused = $100 * \text{diffused volume} / \text{total volume}$).
9. Graphing the data from an experiment gives us a better picture of the surface area to volume relationships we explored. In a typical graph, the independent variable—the variable determined before doing the experiment—is plotted on the X-axis. The dependent variable—the variable or value measured in the experiment—is plotted on the Y-axis. On a graph, plot the surface area of the blocks (the independent variable) on the X-axis in relation to the diffused volume (the dependent variable) on the Y-axis.

	block 1 1x1x1 cm	block 2 2x2x2 cm	block 3 3x3x3 cm	block 4 4x4x4 cm	block 5 1x2x2 cm
Surface area of each block					
Total Volume of each block					
Surface area to volume ratio					
Pink volume remaining after experiment					
average distance diffused					
diffused (colorless) volume					
% diffused volume					

Questions:

1. What do the agar blocks represent? What might the diffused volume represent? What implications might this have for “cells” with different % of diffused volume?
2. In addition to the smallest cube, did any of the other blocks lose all their color? If so, can you explain why?
3. What relationship, if any, does the distance diffused have with either surface area or volume?
4. What relationship does the diffused volume have with surface area?

Exercise 1b - Surface Area to Volume Relationship with Blocks

While you wait for the vinegar to diffuse into the agar blocks, explore surface area to volume relationships using plastic cubes and fill out the first three rows of the table above. Work in pairs. Each group will get 27 plastic cubes with dimensions of 1cm³.

1. Examine a single cube. What is the surface area, volume, and the surface area to volume ratio? (For this exercise, ignore the surface area of the indentations and knobs on the blocks.)
2. Make a cube that is 2cm on each side. What is the surface area, volume, and surface area to volume ratio for this cube?
3. Make a cube that is 3cm on each side. What is the surface area, volume, and surface area to volume ratio?
4. Compare the figures for the three cubes. What is the relationship of the increase in surface area to increases in linear dimension? What is the relationship of the increase in volume to increases in linear dimensions? Compare the surface to volume ratios you calculated in the progressively larger cubes.
5. If the three cubes you constructed represent cells, how might the sizes and the surface area to volume ratio affect their functioning?
6. Using 27 small cubes, build a structure that has the lowest possible surface area to volume relationship.
 What is the surface area? _____
 What is the volume? _____
 What is the surface area to volume ratio? _____
7. Using 27 small cubes, produce a structure that has the largest possible surface area to volume relationship.
 What is the surface area of the structure? _____
 What is the volume of the structure? _____
 What is the surface area to volume ratio? _____
8. Using 27 small cubes, produce a structure that has a large surface area but that is

at most 5 cm long in any one dimension.

What is the surface area of the structure? _____

What is the volume of the structure? _____

What is the surface area to volume ratio? _____

Which of the shapes you produced in steps 6-8 do you think would be most efficient in conserving heat? Why?

Exercise 2: Cooling Rates of Model Organisms

The rate at which an object or organism gains or loses heat is proportional to its surface area. The amount of heat an object or organism can hold is proportional to its volume. Adaptations such as a thick fur covering and a rounded body shape (minimizing surface area in relation to volume) can help an animal survive cold conditions. In colder climates, animals also tend to have shorter legs, ears, tails, and snouts than similar species in warmer climates – again, minimizing surface area in relation to volume. In warm climates, animals tend to have a less rounded shape, smaller size, longer legs and tails, and longer ears that circulate blood near the body surface for cooling (flattened shapes maximize the surface area to volume ratio).

In Exercise 2 we will measure the rate of heat loss in different size “organisms” of the same basic shape.

Materials

- 4 beakers of differing volumes: 250 ml, 400 ml, 600 ml, 1000 mL
- Thermometers (4 per team)
- Rulers
- Graph paper.

Procedure

1. Measure the radius and height of your containers. For the height measurements, use the 200 mL level on the 250 mL beaker, the 300 mL level on the 400 mL beaker, the 500 mL level on the 600 mL beaker, and the 900 mL level on the 1000 mL beaker.
2. Estimate the surface area of each beaker assuming a cylindrical shape: Surface Area = $2(\pi r^2) + (2 \pi r) \times h$. Determine the surface area to volume ratio of your containers. Use the volumes described in Step 1. Record these results of your calculation in the table below.
3. Fill each of your beakers with hot water to the volumes described in Step 1 (200ml in the 250 mL beaker, 300ml in the 400 mL beaker, 500ml in the 600 mL beaker, and 900 mL in the 1000 mL beaker).

4. Arrange the beakers on your bench so that they are spaced several inches apart. Record the temperature of water in each container immediately. Stir the water in each container every minute and record the temperature every 2 minutes for 20 minutes. Record your data in the data table provided below.
5. Graph your data. Again, your graph should have the independent variable, that is, the variable that is pre-determined, on the X-axis, and the dependent variable, the variable that you measure, on the Y-axis. In this case, the independent variable is time and dependent variable is the temperature that you measured. It will be useful to graph all four sets of data on the same graph.
6. Discuss among your group what your graph shows. We want to determine the rate of cooling, that is, the change in temperature over time, or the slope of the line for each of the 4 containers. Keep in mind that as the containers cool and approach room temperature they can't keep on cooling. Thus, choose that portion of the line that has a linear, or constant decrease in temperature to calculate the rate of cooling. Calculate the rate of cooling over this time period for **all four** of your containers.
7. To study the relationship between surface area to volume ratio and the rate of cooling, plot these on a second graph. Remember to put the independent variable on the X-axis and the dependent variable on the Y-axis.

Volume	200 mL	300 mL	500 mL	900 mL
Surface area				
Surface area to Volume Ratio				

Time (min)	0	2	4	6	8	10	12	14	16	18	20
200 mL											
300 mL											
500 mL											
900 mL											

Questions:

1. Which container had the fastest cooling rate? Which had the slowest?
2. How does surface area to volume ratio affect the rate of cooling in your model organisms?